Mechanical Properties

- Elastic deformation
- Plastic deformation
- Fracture
II. Stable Plastic Deformation

- Stress-strain relation is non-linear
- Strain is non-recoverable
  - Relaxation is elastic \( \Rightarrow \) permanent plastic strain
  - Strain is uniform and stable - “work hardening”
- Plasticity initiates at “yield strength”, \( s_y \)
  - In ductile material, \( s_y \) is not obvious
  - \( s_y \) is usually defined by “0.2% offset strain”
    - Yield strength = stress that produces a plastic strain of 0.2%

For a typical ductile metal:
I. Elastic deformation
II. Stable plastic deformation
III. Unstable deformation
IV. Fracture
Forms of the Engineering Stress-Strain Curve

Ductile metal

Brittle solid

Yield point

Elastomer
True Stress and Strain

- True strain is defined from its differential
  \[ d\varepsilon = \frac{dL}{L} = -\frac{dA}{A} \]
  \[ \varepsilon = \int_{L_0}^{L} \frac{dL}{L} \Rightarrow \varepsilon = \ln\left(\frac{L}{L_0}\right) = \ln\left(\frac{A_0}{A}\right) \]
  \[ \varepsilon = \ln\left[\frac{\Delta L}{L_0} + 1\right] \Rightarrow \varepsilon = \ln(1 + e) \]
  - \( \frac{L}{L_0} \) applies when strain is uniform
  - \( \frac{A_0}{A} \) can be used for non-uniform strain

- True stress
  \[ \sigma = \frac{P}{A} = \frac{P}{A_0}\left(\frac{A_0}{A}\right) = s\left(\frac{L}{L_0}\right) \]
  \[ \sigma = s(1 + e) \]

V = constant (plastic deformation):

\[ V = A_0L_0 = AL \]
\[ dV = 0 = AdL + LdA \]
\[ \ldots \Rightarrow \frac{dL}{L} = -\frac{dA}{A} \]

\[ e = \text{engineering strain} \]
\[ e = \frac{\Delta L}{L_0} = \frac{L}{L_0} - 1 \]

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MSE 200A
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Stress-Strain Relations

- Stress-strain relations
  \[ \sigma = s(1 + e) \]
  \[ \varepsilon = \ln(1 + e) \]
  \[ s = \sigma \exp(-\varepsilon) \]
  \[ e = \exp(\varepsilon) - 1 \]
  - stresses equal for small strain

- Why use engineering curve?
  - Clearly shows tensile strength
  - \( s_u \) is an important design value
  - No difference in \( E \), \( s_y = \sigma_y \)
Plastic Deformation: Engineering Significance

- Design: yield strength
  - Almost all structures operate well below yield

- Design: ultimate strength
  - Plastic instability limits the capability of ductile materials

- Manufacture: formability
  - Plasticity is used to form materials into complex shapes

- Service: failure resistance and failure analysis
  - Plasticity provides margin against fracture
  - Deformation patterns record load history
Plastic Deformation Mechanisms

- **Dislocation plasticity** *(our focus in this course)*
  - Dislocation motion causes shear
  - The dominant mechanism of plasticity

- **Diffusion**
  - Atoms diffuse to regions of high stress
  - Most important in high temperature creep *(“diffusional creep”)*

- **Structural phase transformation**
  - Structural transformations can cause shape change *(martensite)*
  - TRIP steel *(“transformation-induced plasticity”)*

- **Grain boundary sliding**
  - Groups of grains tumble by sliding on boundaries
  - “Superplasticity”: rapid creep to deformations of 1000% or more
Example: Indentation of an Al Grain
Indentation Deformation Sequence

\[ b = \frac{a}{2} \langle 110 \rangle \]
The Force on a Dislocation

- Dislocation bounds an area that has slipped
  - Material is sheared by \( b \) when it moves normal to its line
  - The force on the dislocation is \( F = \tau b \)
  - \( \tau \) is the shear stress in the direction of \( b \)
  - Local \( F \) is normal to the line, \textit{whatever} the dislocation shape

- Assume a set of dislocations, \( b \), move an average \(<\delta x>\)
  - If \( \rho \) is the “dislocation density” (line length/unit volume)
  - The strain is \( \gamma = \rho b <\delta x> \)
Plastic Deformation in Shear

- Dislocations cause shear
  - Shear stress required to drive plasticity
  - Hydrostatic stress does not cause plastic deformation

- Yield stress in tension ($\sigma_y$) is determined by critical shear stress ($\tau_c$)
The Critical Resolved Shear Stress

- How does tension (σ) cause shear (τ)
  - Assume tensile stress, σ, along axis
  - Assume glide plane normal at angle, θ
  - Assume Burgers vector at angle, ϕ
  - The “resolved shear stress” is

\[
\tau = \frac{P_b}{A_\theta} = \frac{P \cos(\varphi)}{A \cos(\theta)} = \sigma \cos(\theta) \cos(\varphi)
\]

\[\tau \leq \sigma / 2\]

- Dislocations move when \(\tau = \tau_c\)
  - \(\tau_c\) = “critical resolved shear stress”

- Yield strength, \(\sigma_y\), is such that
  - \(\tau \geq \tau_c\) on the most favorable plane
Yield under Tension

- **Yield strength, $\sigma_y$, is such that**
  - $\tau \geq \tau_c$ on the most favorable plane
  
  \[ \sigma_y = \frac{\tau_c}{\cos(\theta)\cos(\varphi)} \]
  
  $\sigma_y \geq 2\tau_c$
  - Note $\tau_c$ is a material property, not $\sigma_y$

- **Tensile deformation by slip**
  - The slip planes are angled to the bar
  - Slip causes elongation as shown
  - Many slip planes
    \[ \Rightarrow \text{uniform elongation of the bar} \]
Plastic deformation of a polycrystal
- Many grains in all orientations
- Slip in the grain best aligned
- Causes
  - Incremental plastic deformation
  - High stress is adjacent grains
- Deformed regions grow
  - Gradual increase in $\varepsilon_p$
  - Eventual large-scale plasticity

Results
- Gradual yielding
  - Measured by 0.2% offset
- Yield exceeds minimum

\[ \sigma_y = k_T \tau_c \]

($k_T$ = “Taylor factor” $\sim 3$)
Microstructural Control of the Strength

• Add things that inhibit dislocation glide

• Microstructural mechanisms:
  – Crystal structure
    • Make lattice resist dislocation motion (Peierls-Nabarro stress)
  – Refine grain size
    • $\sigma_y \propto d^{-1/2}$
  – Introduce obstacles into grains
    • Solute atoms
    • Other dislocations (work hardening)
    • Precipitates
Inherent Strength: The Peierls-Nabarro Stress

- Lattice resistance to glide
  \[ \tau_p = \frac{2G}{1-\nu} \exp \left[ \frac{2\pi d}{b(1-\nu)} \right] \]
  - \( d = \) distance between slip planes

- Inherently hard materials:
  - High \( G \)
  - Small \( \nu \)
  - Large \((d/b)\) (complex structure)
    - Diamond
    - Si
    - SiO\(_2\)
Grain Refinement

- Grain boundaries resist glide
  - Slip planes not continuous
  - Defect absorption at boundaries

- Hall-Petch relation
  \[ \sigma = \sigma_0 + Kd^{-1/2} \]
  Ex: iron

- Control of grain size
  - Recrystallization in metals
    - Recrystallize and quench
  - Fine-grained powder in ceramics
Ductility Lost at Nanograin Size

Decreasing grain size
⇒ $\sigma_y$ increases
⇒ $\varepsilon_u$ vanishes
Obstacle Hardening

- Force on dislocation:
  - \( f = \tau b \)

- Dislocation bows between obstacles:
  - \( R = \frac{T}{\tau b} \sim \frac{Gb}{2\tau} \)

- Obstacle experiences force:
  - \( f = 2T\cos\left(\frac{\psi}{2}\right) = 2T\beta \)

- When \( \beta = \beta_c \), dislocation cuts through

\[
\tau_c = \frac{2T}{L_b} \beta_c^{3/2} = Gb\beta_c^{3/2} \sqrt{n}
\]

Random distribution of obstacles:

- \( T = \text{line tension} \)
- \( n = \text{obstacle density} \)
- \( \beta = \cos\left(\frac{\psi}{2}\right) \)
Obstacle Hardening

Random distribution of obstacles:

\[ \tau_c = \frac{2T}{L_s b} \beta_c^{3/2} = Gb \beta_c^{3/2} \sqrt{n} \]

- Solute atoms:
  - \( \beta_c \sim 0.01 \)
  - \( n = c \) (concentration)

- Dislocations:
  - \( \beta_c \sim 0.1 \)
  - \( n = \rho \) (dislocation density)

- Precipitates
  - \( \beta_c \sim 0.7 \)
  - \( \sqrt{n} = (1/L_s) \) (obstacle spacing)
Solute Hardening

- Solute have misfit strain
  - Interacts with dislocation field
  - Large solute repels compression
  - Small solute repels tension

- Strength proportional to misfit
  - Interstitial solutes have greatest effect
    - C, N in iron
  - Strength proportional to
    - $c^{1/2}$ when $c$ is small
    - $c^{2/3}$ for larger $c$ (solute overlap)
  - At high $T$, solutes are mobile
    - Strength lost

\[ \sigma_y \propto \sqrt{c} \]

interstitial

substitutional

\[ \sqrt{c} \]
Dislocation Hardening

- Dislocations are
  - Crystallographic obstacles
  - Elastic obstacles

- “Forest dislocations” harden

$$\sigma_y = \alpha G b \sqrt{\rho}$$

$$\alpha \sim 3 \beta_c^{3/2} \sim 0.3$$

- “Work hardening”
  - Dislocations multiply during strain

$$\frac{d\sigma}{d\varepsilon} = \frac{\alpha G b}{2\sqrt{\rho}} \left( \frac{d\rho}{d\varepsilon} \right)$$
Work Hardening

- Dislocations are
  - Crystallographic obstacles
  - Elastic obstacles

- “Forest dislocations” harden
  \[ \sigma_y = \alpha Gb \sqrt{\rho} \]
  \[ \alpha \sim 3 \beta_c^{3/2} \sim 0.3 \]

- “Work hardening”
  - Dislocations multiply during strain
  \[ \frac{d\sigma}{d\varepsilon} = \frac{\alpha Gb}{2\sqrt{\rho}} \left( \frac{d\rho}{d\varepsilon} \right) \]
Precipitation Hardening

Create precipitates

\[ \tau_c = G b \beta_c^{3/2} \sqrt{n} \]

- \( \beta_c \sim 0.5-0.7 \)

“Age hardening”
- Hardens because \( \beta_c \) increases
- Softens when
  - \( \beta_c = \text{max} \) (obstacle impenetrable)
  - \( n \) decreases under coarsening