1. A permanent magnet exhibits a dipole field that can be described by the following equation:

\[ H = \frac{3(\mathbf{m} \cdot \mathbf{r})\hat{r} - r^2 \hat{m}}{r^5} \]

So the magnetic field drops off as \(1/r^3\) at far distances.

(a) We can derive an expression for the magnetic field around a magnetic dipole with magnetic moment \(m\) by beginning with the assumption of the magnetic field \(H=p/r^2\) where \(m\) can be described in terms of fictitious poles with strength \(p\) separated by distance \(l\). Derive the expression for the magnetic field in the limit that \(r \gg l\). Show that

\[ H = \frac{m\sqrt{3} \cos^2 \frac{\pi}{3} + 1}{r^3} \]

(b) Let’s assume permanent magnet A is at the origin with its north pole pointing along the positive y axis. Sketch the field lines surrounding this magnet. What would be the lowest energy state for the orientation of a second permanent magnet B along the y axis and along the x-axis? Draw a diagram of the magnets A & B with moment directions for the two cases. Explain. What is the magnitude of the magnetic field due to A that B feels?

2. Suppose we have a superconducting slab of finite thickness \(d\) as shown below. We apply an external field \(B_a\) parallel to the sides of the slab.

(a) What is the equation that describes the magnetic field distribution inside the superconductor?
(b) What are the boundary conditions?
(c) Solve for an expression for the magnetic field inside the superconductor.
(d) Sketch the magnetic field distribution everywhere for the case that the penetration depth is much smaller than the width of the slab.
(e) What is magnetization per unit area of slab?
3. Analysis of the effective magnetic moment of GdTe$_3$. You will need to use a simple data analysis and graph-plotting package. The data will be available on the website. Or if the website is still not available, it will be available electronically.

In class, we have talked about the susceptibility of paramagnets. GdTe$_3$ is an intermetallic compound. Below its Neel temperature of 12K, it is an antiferromagnet and above this temperature, it exhibits paramagnetism. The magnetism of the material is only due to the localized Gd 4f moments. Below is data for this material.

(a) Use Hund's rules to find the ground state of the Gd$^{3+}$ ion, which has the electron configuration 4f$^7$5s$^2$5p$^6$. Write down L and S, and hence calculate J, g, and the effective magnetic moment.

(b) The susceptibility data shown above were measured in a field of 1000 Oe. Above approximately what temperature would you expect the magnetization to be varying linearly with the applied field and hence the susceptibility to be following Curie's law?

(c) Convert the data into units of emu/mol, where mol refers to 1 mole of GdTe$_3$, and plot the reciprocal susceptibility $\chi^{-1}$ as a function of temperature. Take a linear fit between 50 and 300 K to obtain the Curie constant C, and hence the effective moment in units of $\mu_B$. How does this compare with the theoretical value? Note that interactions between the magnetic moments modify the Curie law such that $\chi = C / (T-q)$, where $q$ is a constant.